

CHAPTER - 2

EQUATIONS





LEARNING OBJECTIVES

After studying this chapter, you will be able to:

- ◆ Understand the concept of equations and its various degrees – linear, simultaneous, quadratic and cubic equations;
- ◆ Know how to solve the different equations using different methods of solution; and
- ◆ Know how to apply equations in co-ordinate geometry.

2.1 INTRODUCTION

Equation is defined to be a mathematical statement of equality. If the equality is true for certain value of the variable involved, the equation is often called a conditional equation and equality sign '=' is used; while if the equality is true for all values of the variable involved, the equation is called an identity.

For Example: $\frac{x+2}{3} + \frac{x+3}{2} = 3$ holds true only for $x=1$.

So it is a conditional. On the other hand, $\frac{x+2}{3} + \frac{x+3}{2} = \frac{5x+13}{6}$

is an identity since it holds for all values of the variable x .

Determination of value of the variable which satisfies an equation is called solution of the equation or root of the equation. An equation in which highest power of the variable is 1 is called a Linear (or a simple) equation. This is also called the equation of degree 1. Two or more linear equations involving two or more variables are called *Simultaneous Linear Equations*. An equation of degree 2 (highest Power of the variable is 2) is called *Quadratic equation* and the equation of degree 3 is called *Cubic Equation*.

For Example: $8x+17(x-3) = 4(4x-9) + 12$ is a Linear equation

$3x^2 + 5x + 6 = 0$ is a quadratic equation.

$4x^3 + 3x^2 + x - 7 = 1$ is a Cubic equation.

$x+2y = 1, 2x+3y = 2$ are jointly called simultaneous equations.

2.2 SIMPLE EQUATION

A simple equation in one unknown x is in the form $ax + b = 0$.

Where a, b are known constants and $a \neq 0$

Note: A simple equation has only one root.

Example: $\frac{4x}{3} - 1 = \frac{14}{15}x + \frac{19}{5}$.

Solution: By transposing the variables in one side and the constants in other side we have



$$\frac{4x}{3} - \frac{14x}{15} = \frac{19}{5} + 1 \quad \text{or} \quad \frac{(20-14)x}{15} = \frac{19+5}{5} \quad \text{or} \quad \frac{6x}{15} = \frac{24}{5}$$

$$x = \frac{24 \times 15}{5 \times 6} = 12$$

Exercise 2 (A)

Choose the most appropriate option (a) (b) (c) or (d)

- The equation $-7x + 1 = 5 - 3x$ will be satisfied for x equal to:
a) 2 b) -1 c) 1 d) none of these
- The Root of the equation $\frac{x+4}{4} + \frac{x-5}{3} = 11$ is
a) 20 b) 10 c) 2 d) none of these
- Pick up the correct value of x for $\frac{x}{30} = \frac{2}{45}$
a) x= 5 b) x=7 c) x=1 $\frac{1}{3}$ d) none of these
- The solution of the equation $\frac{x+24}{5} = 4 + \frac{x}{4}$
a) 6 b) 10 c) 16 d) none of these
- 8 is the solution of the equation
a) $\frac{x+4}{4} + \frac{x-5}{3} = 11$ b) $\frac{x+4}{2} + \frac{x+10}{9} = 8$
c) $\frac{x+24}{5} = 4 + \frac{x}{4}$ d) $\frac{x-15}{10} + \frac{x+5}{5} = 4$
- The value of y that satisfies the equation $\frac{y+11}{6} - \frac{y+1}{9} = \frac{y+7}{4}$ is
a) -1 b) 7 c) 1 d) $-\frac{1}{7}$
- The solution of the equation $(p+2)(p-3) + (p+3)(p-4) = p(2p-5)$ is
a) 6 b) 7 c) 5 d) none of these
- The equation $\frac{12x+1}{4} = \frac{15x-1}{5} + \frac{2x-5}{3x-1}$ is true for
a) x=1 b) x=2 c) x=5 d) x=7



9. Pick up the correct value x for which $\frac{x}{0.5} - \frac{1}{0.05} + \frac{x}{0.005} - \frac{1}{0.0005} = 0$

- a) $x=0$ b) $x=1$ c) $x=10$ d) none of these

Illustrations:

1. The denominator of a fraction exceeds the numerator by 5 and if 3 be added to both the fraction becomes $\frac{3}{4}$. Find the fraction

Let x be the numerator and the fraction be $\frac{x}{x+5}$. By the question $\frac{x+3}{x+5+3} = \frac{3}{4}$ or

$$4x+12 = 3x+24 \text{ or } x = 12$$

The required fraction is $\frac{12}{17}$.

2. If thrice of A's age 6 years ago be subtracted from twice his present age, the result would be equal to his present age. Find A's present age.

Let x years be A's present age. By the question

$$2x - 3(x-6) = x$$

$$\text{or } 2x - 3x + 18 = x$$

$$\text{or } -x + 18 = x$$

$$\text{or } 2x = 18$$

$$\text{or } x = 9$$

\therefore A's present age is 9 years.

3. A number consists of two digits the digit in the ten's place is twice the digit in the unit's place. If 18 be subtracted from the number the digits are reversed. Find the number.

Let x be the digit in the unit's place. So the digit in the ten's place is $2x$. Thus the number becomes $10(2x)+x$. By the question

$$20x+x-18 = 10x + 2x$$

$$\text{or } 21x-18 = 12x$$

$$\text{or } 9x = 18$$

$$\text{or } x = 2$$

So the required number is $10(2 \times 2) + 2 = 42$.

4. For a certain commodity the demand equation giving demand 'd' in kg, for a price 'p' in rupees per kg. is $d = 100(10 - p)$. The supply equation giving the supply s in kg. for a price



p in rupees per kg. is $s = 75(p - 3)$. The market price is such at which demand equals supply. Find the market price and quantity that will be bought and sold.

Given $d = 100(10 - p)$ and $s = 75(p - 3)$.

Since the market price is such that demand (d) = supply (s) we have

$$100(10 - p) = 75(p - 3) \quad \text{or } 1000 - 100p = 75p - 225$$

$$\text{or } -175p = \therefore p = \frac{-1225}{-175} = 7.$$

So market price of the commodity is Rs. 7 per kg.

\therefore the required quantity bought = $100(10 - 7) = 300$ kg.

and the quantity sold = $75(7 - 3) = 300$ kg.

Exercise 2 (B)

Choose the most appropriate option (a) (b) (c) (d)

- The sum of two numbers is 52 and their difference is 2. The numbers are
a) 17 and 15 b) 12 and 10 c) 27 and 25 d) none of these
- The diagonal of a rectangle is 5 cm and one of its sides is 4 cm. Its area is
a) 20 sq.cm. b) 12 sq.cm. c) 10 sq.cm. d) none of these
- Divide 56 into two parts such that three times the first part exceeds one third of the second by 48. The parts are.
a) (20,36) b) (25,31) c) (24,32) d) none of these
- The sum of the digits of a two digit number is 10. If 18 be subtracted from it the digits in the resulting number will be equal. The number is
a) 37 b) 73 c) 75 d) none of these numbers.
- The fourth part of a number exceeds the sixth part by 4. The number is
a) 84 b) 44 c) 48 d) none of these
- Ten years ago the age of a father was four times of his son. Ten years hence the age of the father will be twice that of his son. The present ages of the father and the son are.
a) (50,20) b) (60,20) c) (55,25) d) none of these
- The product of two numbers is 3200 and the quotient when the larger number is divided by the smaller is 2. The numbers are
a) (16,200) b) (160,20) c) (60,30) d) (80,40)
- The denominator of a fraction exceeds the numerator by 2. If 5 be added to the numerator the fraction increases by unity. The fraction is.
a) $\frac{5}{7}$ b) $\frac{1}{3}$ c) $\frac{7}{9}$ d) $\frac{3}{5}$

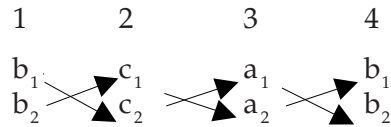


2. Cross Multiplication Method: Let two equations be:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

We write the coefficients of x, y and constant terms and two more columns by repeating the coefficients of x and y as follows:



and the result is given by: $\frac{x}{(b_1c_2 - b_2c_1)} = \frac{y}{(c_1a_2 - c_2a_1)} = \frac{1}{(a_1b_2 - a_2b_1)}$

so the solution is : $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

Example 2: $3x + 2y + 17 = 0$ $5x - 6y - 9 = 0$

Solution: $3x + 2y + 17 = 0$ (i)

$5x - 6y - 9 = 0$ (ii)

Method of elimination: By (i) x3 we get $9x + 6y + 51 = 0$ (iii)

Adding (ii) & (iii) we get $14x + 42 = 0$

$$\text{or } x = -\frac{42}{14} = -3$$

Putting $x = -3$ in (i) we get $3(-3) + 2y + 17 = 0$

$$\text{or, } 2y + 8 = 0 \text{ or, } y = -\frac{8}{2} = -4$$

So $x = -3$ and $y = -4$

Method of cross-multiplication: $3x + 2y + 17 = 0$

$$5x - 6y - 9 = 0$$

$$\frac{x}{2(-9) - 17(-6)} = \frac{y}{17 \times 5 - 3(-9)} = \frac{1}{3(-6) - 5 \times 2}$$

$$\text{or, } \frac{x}{84} = \frac{y}{112} = \frac{1}{-28}$$

$$\text{or } \frac{x}{3} = \frac{y}{4} = \frac{1}{-1}$$

$$\text{or } x = -3 \quad y = -4$$



2.5 METHOD OF SOLVING SIMULTANEOUS LINEAR EQUATION WITH THREE VARIABLES

Example 1: Solve for x , y and z :

$$2x - y + z = 3 \quad x + 3y - 2z = 11 \quad 3x - 2y + 4z = 1$$

Solution: (a) Method of elimination

$$2x - y + z = 3 \quad \dots\dots(i)$$

$$x + 3y - 2z = 11 \quad \dots (ii)$$

$$3x - 2y + 4z = 1 \quad \dots (iii)$$

By (i) $\times 2$ we get

$$4x - 2y + 2z = 6 \quad \dots (iv)$$

By (ii) + (iv), $5x + y = 17 \quad \dots(v)$ [the variable z is thus eliminated]

$$\text{By (ii) } \times 2, \quad 2x + 6y - 4z = 22 \quad \dots(vi)$$

$$\text{By (iii) + (vi), } 5x + 4y = 23 \quad \dots(vii)$$

By (v) - (vii), $-3y = -6$ or $y = 2$

$$\text{Putting } y = 2 \text{ in (v)} \quad 5x + 2 = 17, \text{ or } 5x = 15 \text{ or, } x = 3$$

Putting $x = 3$ and $y = 2$ in (i)

$$2 \times 3 - 2 + z = 3$$

$$\text{or } 6 - 2 + z = 3$$

$$\text{or } 4 + z = 3$$

$$\text{or } z = -1$$

So $x = 3$, $y = 2$, $z = -1$ is the required solution.

(Any two of 3 equations can be chosen for elimination of one of the variables)

(b) Method of cross multiplication

We write the equations as follows:

$$2x - y + (z - 3) = 0$$

$$x + 3y + (-2z - 11) = 0$$

By cross multiplication

$$\frac{x}{-1(-2z - 11) - 3(z - 3)} = \frac{y}{(z - 3) - 2(-2z - 11)} = \frac{1}{2 \times 3 - 1(-1)}$$

$$\frac{x}{20 - z} = \frac{y}{5z + 19} = \frac{1}{7}$$



$$x = \frac{20-z}{7} \quad y = \frac{5z+19}{7}$$

Substituting above values for x and y in equation (iii) i.e. $3x - 2y + yz = 1$, we have

$$3 \left(\frac{20-z}{7} \right) - 2 \left(\frac{5z+19}{7} \right) + 4z = 1$$

$$\text{or } 60 - 3z - 10z - 38 + 28z = 7$$

$$\text{or } 15z = 7 - 22 \text{ or } 15z = -15 \text{ or } z = -1$$

$$\text{Now } x = \frac{20-(-1)}{7} = \frac{21}{7} = 3, \quad y = \frac{5(-1)+19}{7} = \frac{14}{7} = 2$$

Thus $x = 3, y = 2, z = -1$

Example 2: Solve for x, y and z :

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 5, \quad \frac{2}{x} - \frac{3}{y} - \frac{4}{z} = -11, \quad \frac{3}{x} + \frac{2}{y} - \frac{1}{z} = -6$$

Solution: We put $u = \frac{1}{x}, v = \frac{1}{y}, w = \frac{1}{z}$ and get

$$u + v + w = 5 \quad \dots\dots (i)$$

$$2u - 3v - 4w = -11 \quad \dots\dots (ii)$$

$$3u + 2v - w = -6 \quad \dots\dots (iii)$$

$$\text{By (i) + (iii)} \quad 4u + 3v = -1 \quad \dots\dots (iv)$$

$$\text{By (iii) } \times 4 \quad 12u + 8v - 4w = -24 \quad \dots\dots (v)$$

$$\text{By (ii) - (v)} \quad -10u - 11v = 13$$

$$\text{or } 10u + 11v = -13 \quad \dots\dots (vi)$$

$$\text{By (iv) } \times 11 \quad 44u + 33v = -11 \quad \dots\dots (vii)$$

$$\text{By (vi) } \times 3 \quad 30u + 33v = -39 \quad \dots\dots (viii)$$

$$\text{By (vii) - (viii)} \quad 14u = 28 \text{ or } u = 2$$

$$\text{Putting } u = 2 \text{ in (iv)} \quad 4 \times 2 + 3v = -1$$

$$\text{or } 8 + 3v = -1$$

$$\text{or } 3v = -9 \text{ or } v = -3$$

$$\text{Putting } u = 2, v = -3 \text{ in (i) or } 2 - 3 + w = 5$$

$$\text{or } -1 + w = 5 \text{ or } w = 5 + 1 \text{ or } w = 6$$



Thus $x = \frac{1}{u} = \frac{1}{2}$ $y = -\frac{1}{v} = \frac{1}{-3}$ $z = \frac{1}{w} = \frac{1}{6}$ is the solution.

Example 3: Solve for x y and z:

$$\frac{xy}{x+y} = 70, \quad \frac{xz}{x+z} = 84, \quad \frac{yz}{y+z} = 140$$

Solution: We can write as

$$\frac{x+y}{xy} = \frac{1}{70} \quad \text{or} \quad \frac{1}{x} + \frac{1}{y} = \frac{1}{70} \quad \dots\dots (i)$$

$$\frac{x+z}{xz} = \frac{1}{84} \quad \text{or} \quad \frac{1}{z} + \frac{1}{x} = \frac{1}{84} \quad \dots\dots (ii)$$

$$\frac{y+z}{yz} = \frac{1}{140} \quad \text{or} \quad \frac{1}{y} + \frac{1}{z} = \frac{1}{140} \quad \dots\dots (iii)$$

By (i) + (ii) + (iii), we get $2 \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right) = \frac{1}{70} + \frac{1}{84} + \frac{1}{140} = \frac{14}{420}$

or $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{7}{420} = \frac{1}{60}$ (iv)

By (iv)-(iii) $\frac{1}{x} = \frac{1}{60} - \frac{1}{140} = \frac{4}{420}$ or $x = 105$

By (iv)-(ii) $\frac{1}{y} = \frac{1}{60} - \frac{1}{84} = \frac{2}{420}$ or $y = 210$

By (iv)-(i) $\frac{1}{z} = \frac{1}{60} - \frac{1}{70}$ or $z = 420$

Required solution is $x = 105, y = 210, z = 420$

Exercise 2 (C)

Choose the most appropriate option (a) (b) (c) (d)

- The solution of the set of equations $3x + 4y = 7, 4x - y = 3$ is
 a) (1, -1) b) (1, 1) c) (2, 1) d) (1, -2)
- The values of x and y satisfying the equations $\frac{x}{2} + \frac{y}{3} = 2, x + 2y = 8$ are given by the pair.
 a) (3, 2) b) (-2, -3) c) (2, 3) d) none of these



3. $\frac{x}{p} + \frac{y}{q} = 2$, $x + y = p + q$ are satisfied by the values given by the pair.
a) $(x=p, y=q)$ b) $(x=q, y=p)$ c) $(x=1, y=1)$ d) none of these
4. The solution for the pair of equations $\frac{1}{16x} + \frac{1}{15y} = \frac{9}{20}$, $\frac{1}{20x} - \frac{1}{27y} = \frac{4}{45}$ is given by
(a) $\left(\frac{1}{4}, \frac{1}{3}\right)$ (b) $\left(\frac{1}{3}, \frac{1}{4}\right)$ (c) (3 4) (d) (4 3)
5. Solve for x and y: $\frac{4}{x} - \frac{5}{y} = \frac{x+y}{xy} + \frac{3}{10}$ and $3xy = 10(y-x)$. The values of x and y are given by the pair.
a) (5, 2) b) (-2, -5) c) (2, -5) d) (2, 5)
6. The pair satisfying the equations $x + 5y = 36$, $\frac{x+y}{x-y} = \frac{5}{3}$ is given by
a) (16, 4) b) (4, 16) c) (4, 8) d) none of these.
7. Solve for x and y : $x-3y = 0$, $x+2y = 20$. The values of x and y are given as
a) $x=4, y=12$ b) $x=12, y=4$ c) $x=5, y=4$ d) none of these
8. The simultaneous equations $7x-3y = 31$, $9x-5y = 41$ have solutions given by
a) (-4, -1) b) (-1, 4) c) (4, -1) d) (3, 7)
9. $1.5x + 2.4y = 1.8$, $2.5(x+1) = 7y$ have solutions as
a) (0.5, 0.4) b) (0.4, 0.5) c) $\left(\frac{1}{2}, \frac{2}{5}\right)$ d) (2, 5)
10. The values of x and Y satisfying the equations $\frac{3}{x+y} + \frac{2}{x-y} = 3$, $\frac{2}{x+y} + \frac{3}{x-y} = 3\frac{2}{3}$ are given by
a) (1, 2) b) (-1, -2) c) $\left(1, \frac{1}{2}\right)$ d) (2, 1)



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Exercise 2 (D)

Choose the most appropriate option (a) (b) (c) (d) as the solution to the given set of equations :

- $1.5x + 3.6y = 2.1, 2.5(x+1) = 6y$
a) (0.2, 0.5) b) (0.5, 0.2) c) (2, 5) d) (-2, -5)
- $2. \frac{x}{5} + \frac{y}{6} + 1 = \frac{x}{6} + \frac{y}{5} = 28$
a) (6, 9) b) (9, 6) c) (60, 90) d) (90, 60)
- $3. \frac{x}{4} = \frac{y}{3} = \frac{z}{2} \quad 7x + 8y + 5z = 62$
a) (4, 3, 2) b) (2, 3, 4) c) (3, 4, 2) d) (4, 2, 3)
- $4. \frac{x}{x+y} = 20, \frac{y}{y+z} = 40, \frac{z}{z+x} = 24$
a) (120, 60, 30) b) (60, 30, 120) c) (30, 120, 60) d) (30, 60, 120)
- $5. 2x + 3y + 4z = 0, x + 2y - 5z = 0, 10x + 16y - 6z = 0$
a) (0,0,0) b) (1, -1, 1) c) (3, 2, -1) d) (1, 0, 2)
- $6. \frac{1}{3}(x+y) + 2z = 21, 3x - \frac{1}{2}(y+z) = 65, x + \frac{1}{2}(x+y-z) = 38$
a) (4,9,5) b) (2,9,5) c) (24, 9, 5) d) (5, 24, 9)
- $7. \frac{4}{x} - \frac{5}{y} = \frac{x+y}{xy} + \frac{3}{10} \quad 3xy = 10(y-x)$
a) (2, 5) b) (5, 2) c) (2, 7) d) (3, 4)
- $8. \frac{x}{0.01} + \frac{y+0.03}{0.05} = \frac{y}{0.02} + \frac{x+0.03}{0.04} = 2$
a) (1, 2) b) (0.1, 0.2) c) (0.01, 0.02) d) (0.02, 0.01)
- $9. \frac{x}{y-x} = 110, \frac{y}{z-y} = 132, \frac{z}{z+x} = \frac{60}{11}$
a) (12, 11, 10) b) (10, 11, 12) c) (11, 10, 12) d) (12, 10, 11)
- $10. 3x-4y+70z = 0, 2x+3y-10z = 0, x+2y+3z = 13$
a) (1, 3, 7) b) (1, 7, 3) c) (2, 4, 3) d) (-10, 10, 1)



2.6 PROBLEMS LEADING TO SIMULTANEOUS EQUATIONS

Illustrations :

1. If the numerator of a fraction is increased by 2 and the denominator by 1 it becomes 1. Again if the numerator is decreased by 4 and the denominator by 2 it becomes $\frac{1}{2}$. Find the fraction

Solution: Let $\frac{x}{y}$ be the required fraction.

By the question $\frac{x+2}{y+1}=1, \frac{x-4}{y-2}=\frac{1}{2}$

Thus $x + 2 = y + 1$ or $x - y = -1$ (i)

and $2x - 8 = y - 2$ or $2x - y = 6$ (ii)

By (i) - (ii) $-x = -7$ or $x = 7$

from (i) $7 - y = -1$ or $y = 8$

So the required fraction is $\frac{7}{8}$.

2. The age of a man is three times the sum of the ages of his two sons and 5 years hence his age will be double the sum of their ages. Find the present age of the man?

Solution: Let x years be the present age of the man and sum of the present ages of the two sons be y years.

By the condition $x = 3y$ (i)

and $x + 5 = 2(y + 5)$ (ii)

From (i) & (ii) $3y + 5 = 2(y + 5)$

or $3y + 5 = 2y + 10$

or $3y - 2y = 10 - 5$

or $y = 5$

$x = 3 \times y = 3 \times 5 = 15$

Hence the present age of the man is 15 years

3. A number consist of three digit of which the middle one is zero and the sum of the other digits is 9. The number formed by interchanging the first and third digits is more than the original number by 297 find the number.

Solution: Let the number be $100x + y$. we have $x + y = 9$(i)

Also $100y + x = 100x + y + 297$ (ii)

From (ii) $99(x - y) = -297$

or $x - y = -3$ (iii)



Adding (i) and (ii) $2x = 6$ $x = 3$ \therefore from (i) $y = 6$

\therefore Hence the number is 306.

Exercise 2 (E)

Choose the most appropriate option (a) (b) (c) (d)

- Monthly incomes of two persons are in the ratio 4 : 5 and their monthly expenses are in the ratio 7 : 9. If each saves Rs. 50 per month find their monthly incomes.
a) (500, 400) b) (400, 500) c) (300, 600) d) (350, 550)
- Find the fraction which is equal to $\frac{1}{2}$ when both its numerator and denominator are increased by 2. It is equal to $\frac{3}{4}$ when both are increased by 12.
a) $\frac{3}{8}$ b) $\frac{5}{8}$ c) $\frac{3}{8}$ d) $\frac{2}{3}$
- The age of a person is twice the sum of the ages of his two sons and five years ago his age was thrice the sum of their ages. Find his present age.
a) 60 years b) 52 years c) 51 years d) 50 years.
- A number between 10 and 100 is five times the sum of its digits. If 9 be added to it the digits are reversed find the number.
a) 54 b) 53 c) 45 d) 55
- The wages of 8 men and 6 boys amount to Rs. 33. If 4 men earn Rs. 4.50 more than 5 boys determine the wages of each man and boy.
a) (Rs. 1.50, Rs. 3) b) (Rs. 3, Rs. 1.50)
c) (Rs. 2.50, Rs. 2) d) (Rs. 2, Rs. 2.50)
- A number consisting of two digits is four times the sum of its digits and if 27 be added to it the digits are reversed. The number is :
a) 63 b) 35 c) 36 d) 60
- Of two numbers, $\frac{1}{5}$ th of the greater is equal to $\frac{1}{3}$ rd of the smaller and their sum is 16. The numbers are:
a) (6, 10) b) (9, 7) c) (12, 4) d) (11, 5)
- Y is older than x by 7 years 15 years back X's age was $\frac{3}{4}$ of Y's age. Their present ages are:
a) (X=36, Y=43) b) (X=50, Y=43)
c) (X=43, Y=50) d) (X=40, Y=47)
- The sum of the digits in a three digit number is 12. If the digits are reversed the number is increased by 495 but reversing only of the ten's and unit digits in creases the number by 36. The number is
a) 327 b) 372 c) 237 d) 273



10. Two numbers are such that twice the greater number exceeds twice the smaller one by 18 and $\frac{1}{3}$ of the smaller and $\frac{1}{5}$ of the greater number are together 21. The numbers are :
- a) (36, 45) b) (45, 36) c) (50, 41) d) (55, 46)
11. The demand and supply equations for a certain commodity are $4q + 7p = 17$ and $p = \frac{q}{3} + \frac{7}{4}$, respectively where p is the market price and q is the quantity then the equilibrium price and quantity are:
- (a) $2, \frac{3}{4}$ (b) $3, \frac{1}{2}$ (c) $5, \frac{3}{5}$ (d) None of these.

2.7 QUADRATIC EQUATION

An equation of the form $ax^2 + bx + c = 0$ where x is a variable and a, b, c are constants with $a \neq 0$ is called a quadratic equation or equation of the second degree.

When $b=0$ the equation is called a pure quadratic equation; when $b \neq 0$ the equation is called an affected quadratic.

- Examples:** i) $2x^2 + 3x + 5 = 0$
ii) $x^2 - x = 0$
iii) $5x^2 - 6x - 3 = 0$

The value of the variable say x is called the root of the equation. A quadratic equation has got two roots.

How to find out the roots of a quadratic equation:

$$ax^2 + bx + c = 0 \quad (a \neq 0)$$

$$\text{or } x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\text{or } x^2 + 2\frac{b}{2a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\text{or } \left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\text{or } x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$\text{or } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



EQUATIONS

Let one root be α and the other root be β

$$\begin{aligned} \text{Now } \alpha + \beta &= \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2b}{2a} = \frac{-b}{a} \end{aligned}$$

$$\text{Thus sum of roots} = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Next } \alpha\beta = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a} \right) \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a} \right) = \frac{c}{a}$$

$$\text{So the product of the roots} = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

2.8 HOW TO CONSTRUCT A QUADRATIC EQUATION

For the equation $ax^2 + bx + c = 0$ we have

$$\text{or } x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\text{or } x^2 - \left(-\frac{b}{a} \right)x + \frac{c}{a} = 0$$

$$\text{or } x^2 - (\text{Sum of the roots})x + \text{Product of the roots} = 0$$

2.9 NATURE OF THE ROOTS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- i) If $b^2 - 4ac = 0$ the roots are real and equal;
- ii) If $b^2 - 4ac > 0$ then the roots are real and unequal (or distinct);
- iii) If $b^2 - 4ac < 0$ then the roots are imaginary;
- iv) If $b^2 - 4ac$ is a perfect square ($\neq 0$) the roots are real, rational and unequal (distinct);
- v) If $b^2 - 4ac$ but not a perfect square the roots are real, irrational and unequal.

Since $b^2 - 4ac$ discriminates the roots $b^2 - 4ac$ is called the discriminant in the equation $ax^2 + bx + c = 0$ as it actually discriminates between the roots.



Note: (a) Irrational roots occur in pairs that is if $(m + \sqrt{n})$ is a root then $(m - \sqrt{n})$ is the other root of the same equation.

(b) If one root is reciprocal to the other root then their product is 1 and so $\frac{c}{a} = 1$
i.e. $c = a$

(c) If one root is equal to other root but opposite in sign then.

their sum = 0 and so $\frac{b}{a} = 0$. i.e. $b = 0$.

Example 1 : Solve $x^2 - 5x + 6 = 0$

Solution: 1st method : $x^2 - 5x + 6 = 0$

$$\text{or } x^2 - 2x - 3x + 6 = 0$$

$$\text{or } x(x-2) - 3(x-2) = 0$$

$$\text{or } (x-2)(x-3) = 0$$

$$\text{or } x = 2 \text{ or } 3$$

2nd method (By formula) $x^2 - 5x + 6 = 0$

Here $a = 1$ $b = -5$ $c = 6$ (comparing the equation with $ax^2 + bx + c = 0$)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{25 - 24}}{2}$$

$$= \frac{5 \pm 1}{2} = \frac{6}{2} \text{ and } \frac{4}{2}, \therefore x = 3 \text{ and } 2$$

Example 2: Examine the nature of the roots of the following equations.

i) $x^2 - 8x + 16 = 0$

ii) $3x^2 - 8x + 4 = 0$

iii) $5x^2 - 4x + 2 = 0$

iv) $2x^2 - 6x - 3 = 0$

Solution: (i) $a = 1$ $b = -8$ $c = 16$

$$b^2 - 4ac = (-8)^2 - 4.1.16 = 64 - 64 = 0$$

The roots are real and equal.

(ii) $3x^2 - 8x + 4 = 0$

$$a = 3 \quad b = -8 \quad c = 4$$

$$b^2 - 4ac = (-8)^2 - 4.3.4 = 64 - 48 = 16 > 0 \text{ and a perfect square}$$

The roots are real, rational and unequal



EQUATIONS

$$(iii) \quad 5x^2 - 4x + 2 = 0$$

$$b^2 - 4ac = (-4)^2 - 4.5.2 = 16 - 40 = -24 < 0$$

The roots are imaginary and unequal

$$(iv) \quad 2x^2 - 6x - 3 = 0$$

$$b^2 - 4ac = (-6)^2 - 4.2(-3)$$

$$= 36 + 24 = 60 > 0$$

The roots are real and unequal. Since $b^2 - 4ac$ is not a perfect square the roots are real irrational and unequal.

Illustrations:

1. If α and β be the roots of $x^2 + 7x + 12 = 0$ find the equation whose roots are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$.

Solution : Now sum of the roots of the required equation

$$= (\alpha + \beta)^2 + (\alpha - \beta)^2 = (-7)^2 + (\alpha + \beta)^2 - 4\alpha\beta$$

$$= 49 + (-7)^2 - 4 \times 12$$

$$= 49 + 49 - 48 = 50$$

$$\begin{aligned} \text{Product of the roots of the required equation} &= (\alpha - \beta)^2 \cdot (\alpha - \beta)^2 \\ &= 49(49 - 48) = 49 \end{aligned}$$

Hence the required equation is

$$x^2 - (\text{sum of the roots})x + \text{product of the roots} = 0$$

$$\text{or } x^2 - 50x + 49 = 0$$

2. If α, β be the roots of $2x^2 - 4x - 1 = 0$ find the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

$$\text{Solution: } \alpha + \beta = \frac{-(-4)}{2} = 2, \quad \alpha\beta = \frac{-1}{2}$$

$$\therefore \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta}$$

$$\frac{2^3 - 3\left(-\frac{1}{2}\right) \cdot 2}{\left(-\frac{1}{2}\right)} = -22$$



3. Solve $x : 4^x - 3 \cdot 2^{x+2} + 2^5 = 0$

Solution: $4^x - 3 \cdot 2^{x+2} + 2^5 = 0$

or $(2^x)^2 - 3 \cdot 2^x \cdot 2^2 + 32 = 0$

or $(2^x)^2 - 12 \cdot 2^x + 32 = 0$

or $y^2 - 12y + 32 = 0$ (taking $y = 2^x$)

or $y^2 - 8y - 4y + 32 = 0$

or $y(y - 8) - 4(y - 8) = 0 \quad \therefore (y - 8)(y - 4) = 0$

either $y - 8 = 0$ or $y - 4 = 0 \quad \therefore y = 8$ or $y = 4$.

$\Rightarrow 2^x = 8 = 2^3$ or $2^x = 4 = 2^2 \Rightarrow x = 3$ or $x = 2$.

4. Solve $\left(x - \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) = 7\frac{1}{4}$.

Solution: $\left(x - \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) = 7\frac{1}{4}$

$\left(x - \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) = \frac{29}{4}$

or $\left(x + \frac{1}{x}\right)^2 - 4 + 2\left(x + \frac{1}{x}\right) = \frac{29}{4}$

[as $(a - b)^2 = (a + b)^2 - 4ab$]

or $p^2 + 2p - \frac{45}{4} = 0$ Taking $p = x + \frac{1}{x}$

or $4p^2 + 8p - 45 = 0$

or $4p^2 + 18p - 10p - 45 = 0$

or $2p(2p + 9) - 5(2p + 9) = 0$

or $(2p - 5)(2p + 9) = 0$.

\therefore Either $2p + 9 = 0$ or $2p - 5 = 0 \Rightarrow p = -\frac{9}{2}$ or $p = \frac{5}{2}$

\therefore Either $x + \frac{1}{x} = -\frac{9}{2}$ or $x + \frac{1}{x} = \frac{5}{2}$

i.e. Either $2x^2 + 9x + 2 = 0$ or $2x^2 - 5x + 2 = 0$

i.e. Either $x = \frac{-9 \pm \sqrt{81 - 16}}{4}$ or $x = \frac{5 \pm \sqrt{25 - 16}}{4}$



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i.e. Either $x = \frac{-9 \pm \sqrt{65}}{4}$ or $x = 2\frac{1}{2}$.

5. Solve $2^{x-2} + 2^{3-x} = 3$

Solution: $2^{x-2} + 2^{3-x} = 3$

or $2^x \cdot 2^{-2} + 2^3 \cdot 2^{-x} = 3$

or $\frac{2^x}{2^2} + \frac{2^3}{2^x} = 3$

or $\frac{t}{4} + \frac{8}{t} = 3$ when $t = 2^x$

or $t^2 + 32 = 12t$

or $t^2 - 12t + 32 = 0$

or $t^2 - 8t - 4t + 32 = 0$

or $t(t-8) - 4(t-8) = 0$

or $(t-4)(t-8) = 0$

$\therefore t = 4, 8$

For $t = 4$ $2^x = 4 = 2^2$ i.e. $x = 2$

For $t = 8$ $2^x = 8 = 2^3$ i.e. $x = 3$

6. If one root of the equation is $2 - \sqrt{3}$ form the equation.

Solution: other roots is $2 + \sqrt{3}$ \therefore sum of two roots $= 2 - \sqrt{3} + 2 + \sqrt{3} = 4$

Product of roots $= (2 - \sqrt{3})(2 + \sqrt{3}) = 4 - 3 = 1$

\therefore Required equation is : $x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

or $x^2 - 4x + 1 = 0$.

7. If α, β are the two roots of the equation $x^2 - px + q = 0$ form the equation

whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

Solution: As α, β are the roots of the equation $x^2 - px + q = 0$

$\alpha + \beta = -(-p) = p$ and $\alpha\beta = q$.

Now $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{p^2 - 2q}{q}$; and $\frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$



$$\therefore \text{Required equation is } x^2 - \left(\frac{p^2 - 2q}{q} \right) x + 1 = 0$$

$$\text{or } qx^2 - (p^2 - 2q)x + q = 0$$

8. If the roots of the equation $p(q - r)x^2 + q(r - p)x + r(p - q) = 0$

are equal show that $\frac{2}{q} = \frac{1}{p} + \frac{1}{r}$.

Solution: Since the roots of the given equation are equal the discriminant must be zero i.e. $q^2(r - p)^2 - 4 \cdot p(q - r) \cdot r(p - q) = 0$

$$\text{or } q^2 r^2 + q^2 p^2 - 2q^2 rp - 4pr(pq - pr - q^2 + qr) = 0$$

$$\text{or } p^2 q^2 + q^2 r^2 + 4p^2 r^2 + 2q^2 pr - 4p^2 qr - 4pqr^2 = 0$$

$$\text{or } (pq + qr - 2rp)^2 = 0$$

$$\therefore pq + qr = 2pr$$

$$\text{or } \frac{p}{2pr} = 1 \quad \text{or, } \frac{q}{2} \cdot \frac{(p+r)}{pr} = 1 \quad \text{or, } \frac{1}{r} + \frac{1}{p} = \frac{2}{q}$$

Exercise 2(F)

Choose the most appropriate option (a) (b) (c) (d)

1. If the roots of the equation $2x^2 + 8x - m^3 = 0$ are equal then value of m is

- (a) -3 (b) -1 (c) 1 (d) -2

2. If $2^{2x+3} - 3^2 \cdot 2^x + 1 = 0$ then values of x are

- (a) 0, 1 (b) 1, 2 (c) 0, 3 (d) 0, -3

3. The values of $4 + \frac{1}{4 + \frac{1}{4 + \frac{1}{4 + \dots 2}}}$

- (a) $1 \pm \sqrt{2}$ (b) $2 \pm \sqrt{5}$ (c) $2 \pm \sqrt{3}$ (d) none of these

4. If $\alpha\beta$ be the roots of the equation $2x^2 - 4x - 3 = 0$

the value of $\alpha^2 + \beta^2$ is

- a) 5 b) 7 c) 3 d) -4



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5. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals then $\frac{b^2}{ac} + \frac{bc}{a^2}$ is equal to
- a) 2 b) -2 c) 1 d) -1
6. The equation $x^2 - (p+4)x + 2p + 5 = 0$ has equal roots the values of p will be.
- a) ± 1 b) 2 c) ± 2 d) -2
7. The roots of the equation $x^2 + (2p-1)x + p^2 = 0$ are real if.
- a) $p \geq 1$ b) $p \leq -4$ c) $p \geq 1/4$ d) $p \leq 1/4$
8. If $x = m$ is one of the solutions of the equation $2x^2 + 5x - m = 0$ the possible values of m are
- a) (0, 2) b) (0, -2) c) (0, 1) d) (1, -1)
9. If p and q are the roots of $x^2 + 2x + 1 = 0$ then the values of $p^2 + q^2$ becomes
- a) 2 b) -2 c) 4 d) -4
10. If $L + M + N = 0$ and L, M, N are rationals the roots of the equation $(M+N-L)x^2 + (N+L-M)x + (L+M-N) = 0$ are
- a) real and irrational b) real and rational
c) imaginary and equal d) real and equal
11. If α and β are the roots of $x^2 = x+1$ then value of $\frac{\alpha^2}{\beta} - \frac{\beta^2}{\alpha}$ is
- a) $2\sqrt{5}$ b) $\sqrt{5}$ c) $3\sqrt{5}$ d) $-2\sqrt{5}$
12. If $p \neq q$ and $p^2 = 5p - 3$ and $q^2 = 5q - 3$ the equation having roots as $\frac{p}{q}$ and $\frac{q}{p}$ is
- a) $x^2 - 19x + 3 = 0$ b) $3x^2 - 19x - 3 = 0$
c) $3x^2 - 19x + 3 = 0$ d) $3x^2 + 19x + 3 = 0$
13. If one root of $5x^2 + 13x + p = 0$ be reciprocal of the other then the value of p is
- a) -5 b) 5 c) $1/5$ d) $-1/5$

Exercise 2 (G)

Choose the most appropriate option (a) (b) (c) (d)

1. A solution of the quadratic equation $(a+b-2c)x^2 + (2a-b-c)x + (c+a-2b) = 0$ is
- a) $x = 1$ b) $x = -1$ c) $x = 2$ d) $x = -2$
2. If the root of the equation $x^2 - 8x + m = 0$ exceeds the other by 4 then the value of m is
- a) $m = 10$ b) $m = 11$ c) $m = 9$ d) $m = 12$



3. The values of x in the equation $7(x+2p)^2 + 5p^2 = 35xp + 117p^2$ are
a) $(4p, -3p)$ b) $(4p, 3p)$ c) $(-4p, 3p)$ d) $(-4p, -3p)$
4. The solutions of the equation $\frac{6x}{x+1} + \frac{6(x+1)}{x} = 13$ are
a) $(2, 3)$ b) $(3, -2)$ c) $(-2, -3)$ d) $(2, -3)$
5. The satisfying values of x for the equation $\frac{1}{x+p+q} = \frac{1}{x} + \frac{1}{p} + \frac{1}{q}$ are
a) (p, q) b) $(-p, -q)$ c) $(p, -p)$ d) $(-p, q)$
6. The values of x for the equation $x^2+9x+18 = 6-4x$ are
a) $(1, 12)$ b) $(-1, -12)$ c) $(1, -12)$ d) $(-1, 12)$
7. The values of x satisfying the equation $\sqrt{(2x^2+5x-2)} - \sqrt{(2x^2+5x-9)} = 1$ are
a) $(2, -9/2)$ b) $(4, -9)$ c) $(2, 9/2)$ d) $(-2, 9/2)$
8. The solution of the equation $3x^2-17x + 24 = 0$ are
a) $(2, 3)$ b) $(2, 3\frac{2}{3})$ c) $(3, 2\frac{2}{3})$ d) $(3, \frac{2}{3})$
9. The equation $\frac{3(3x^2+15)}{6} + 2x^2 + 9 = \frac{2x^2+96}{7} + 6$ has got the solution as
a) $(1, 1)$ b) $(1/2, -1)$ c) $(1, -1)$ d) $(2, -1)$
10. The equation $\left(\frac{\lambda-m}{2}\right)x^2 - \left(\frac{\lambda+m}{2}\right)x + m = 0$ has got two values of x to satisfy the equation given as
a) $\left(1, \frac{2m}{l-m}\right)$ b) $\left(1, \frac{m}{l-m}\right)$ c) $\left(1, \frac{2l}{l-m}\right)$ d) $\left(1, \frac{l}{l-m}\right)$

2.10 PROBLEMS ON QUADRATIC EQUATION

1. Difference between a number and its positive square root is 12; find the numbers?

Solution: Let the number be x .

Then $x - \sqrt{x} = 12$ (i)



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$$(\sqrt{x})^2 - \sqrt{x} - 12 = 0. \quad \text{Taking } y = \sqrt{x}, y^2 - y - 12 = 0$$

$$\text{or } (y - 4)(y + 3) = 0 \quad \therefore \text{Either } y = 4 \text{ or } y = -3 \quad \text{i.e. Either } \sqrt{x} = 4 \quad \text{or } \sqrt{x} = -3$$

$$\text{If } \sqrt{x} = -3 \quad x = 9 \text{ if does not satisfy equation (i)} \quad \text{so } \sqrt{x} = 4 \text{ or } x = 16.$$

2. A piece of iron rod costs Rs. 60. If the rod was 2 metre shorter and each metre costs Re. 1.00 more, the cost would remain unchanged. What is the length of the rod?

Solution: Let the length of the rod be x metres. The rate per meter is Rs. $\frac{60}{x}$.

New Length = $(x - 2)$; as the cost remain the same the new rate per meter is $\frac{60}{x-2}$

$$\text{As given } \frac{60}{x-2} = \frac{60}{x} + 1$$

$$\text{or } \frac{60}{x-2} - \frac{60}{x} = 1$$

$$\text{or } \frac{120}{x(x-2)} = 1$$

$$\text{or } x^2 - 2x = 120$$

$$\text{or } x^2 - 2x - 120 = 0 \quad \text{or } (x - 12)(x + 10) = 0.$$

Either $x = 12$ or $x = -10$ (not possible)

\therefore Hence the required length = 12m.

3. Divide 25 into two parts so that sum of their reciprocals is $\frac{1}{6}$.

Solution: let the parts be x and $25 - x$

$$\text{By the question } \frac{1}{x} + \frac{1}{25-x} = \frac{1}{6}$$

$$\text{or } \frac{25-x+x}{x(25-x)} = \frac{1}{6}$$

$$\text{or } 150 = 25x - x^2$$

$$\text{or } x^2 - 25x + 150 = 0$$

$$\text{or } x^2 - 15x - 10x + 150 = 0$$

$$\text{or } x(x-15) - 10(x-15) = 0$$

$$\text{or } (x-15)(x-10) = 0$$



or $x = 10, 15$

So the parts of 25 are 10 and 15.

Exercise 2 (H)

Choose the most appropriate option (a) (b) (c) (d)

1. The sum of two numbers is 8 and the sum of their squares is 34. Taking one number as x form an equation in x and hence find the numbers. The numbers are
a) (7, 10) b) (4, 4) c) (3, 5) d) (2, 6)
2. The difference of two positive integers is 3 and the sum of their squares is 89. Taking the smaller integer as x form a quadratic equation and solve it to find the integers. The integers are.
a) (7, 4) b) (5, 8) c) (3, 6) d) (2, 5)
3. Five times of a positive whole number is 3 less than twice the square of the number. The number is
a) 3 b) 4 c) -3 d) 2
4. The area of a rectangular field is 2000 sq.m and its perimeter is 180m. Form a quadratic equation by taking the length of the field as x and solve it to find the length and breadth of the field. The length and breadth are
a) (205m, 80m) b) (50m, 40m) c) (40m, 50m) d) none
5. Two squares have sides p cm and $(p + 5)$ cms. The sum of their squares is 625 sq. cm. The sides of the squares are
(a) (10 cm, 30 cm) (b) (12 cm, 25 cm)
(c) (15 cm, 20 cm) (d) none of these
6. Divide 50 into two parts such that the sum of their reciprocals is $1/12$. The numbers are
a) (24, 26) b) (28, 22) c) (27, 23) d) (20, 30)
7. There are two consecutive numbers such that the difference of their reciprocals is $1/240$. The numbers are
(a) (15, 16) (b) (17, 18) (c) (13, 14) (d) (12, 13)
8. The hypotenuse of a right-angled triangle is 20cm. The difference between its other two sides be 4cm. The sides are
(a) (11cm, 15cm) (b) (12cm, 16cm) (c) (20cm, 24cm) (d) none of these
9. The sum of two numbers is 45 and the mean proportional between them is 18. The numbers are
a) (15, 30) b) (32, 13) c) (36, 9) d) (25, 20)
10. The sides of an equilateral triangle are shortened by 12 units 13 units and 14 units respectively and a right angle triangle is formed. The side of the equilateral triangle is
(a) 17 units (b) 16 units (c) 15 units (d) 18 units



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11. A distributor of apple Juice has 5000 bottle in the store that it wishes to distribute in a month. From experience it is known that demand D (in number of bottles) is given by $D = -2000p^2 + 2000p + 17000$. The price per bottle that will result zero inventory is
 (a) Rs. 3 (b) Rs. 5 (c) Rs. 2 (d) none of these.
12. The sum of two irrational numbers multiplied by the larger one is 70 and their difference is multiplied by the smaller one is 12; the two numbers are
 (a) $3\sqrt{2}, 2\sqrt{3}$ (b) $5\sqrt{2}, 3\sqrt{5}$ (c) $2\sqrt{2}, 5\sqrt{2}$ (d) none of these.

2.11 SOLUTION OF CUBIC EQUATION

On trial basis putting some value of x to check whether LHS is zero then to get a factor. This is a trial and error method. With this factor to factorise the LHS and then to get values of x .

Illustrations :

1. Solve $x^3 - 7x + 6 = 0$

Putting $x = 1$ L.H.S is Zero. So $(x-1)$ is a factor of $x^3 - 7x + 6$

We write $x^3 - 7x + 6 = 0$ in such a way that $(x-1)$ becomes its factor. This can be achieved by writing the equation in the following form.

$$\text{or } x^3 - x^2 + x^2 - x - 6x + 6 = 0$$

$$\text{or } x^2(x-1) + x(x-1) - 6(x-1) = 0$$

$$\text{or } (x-1)(x^2 + x - 6) = 0$$

$$\text{or } (x-1)(x^2 + 3x - 2x - 6) = 0$$

$$\text{or } (x-1)\{ x(x+3) - 2(x+3) \} = 0$$

$$\text{or } (x-1)(x-2)(x+3) = 0$$

$$\therefore \text{ or } x = 1, 2, -3$$

2. Solve for real x : $x^3 + x + 2 = 0$

Solution: By trial we find that $x = -1$ makes the LHS zero. So $(x + 1)$ is a factor of $x^3 + x + 2$

We write $x^3 + x + 2 = 0$ as $x^3 + x^2 - x^2 - x + 2x + 2 = 0$

$$\text{or } x^2(x + 1) - x(x + 1) + 2(x + 1) = 0$$

$$\text{or } (x + 1)(x^2 - x + 2) = 0.$$

$$\text{Either } x + 1 = 0$$

$$\text{or } x^2 - x + 2 = 0 \text{ i.e. } x = -1$$

$$\text{i.e. } x = \frac{1 \pm \sqrt{1-8}}{2} = \frac{1 \pm \sqrt{-7}}{2}$$



As $x = \frac{1 \pm \sqrt{-7}}{2}$ is not real, $x = -1$ is the required solution.

Exercise 2 (I)

Choose the most appropriate option (a) (b) (c) (d)

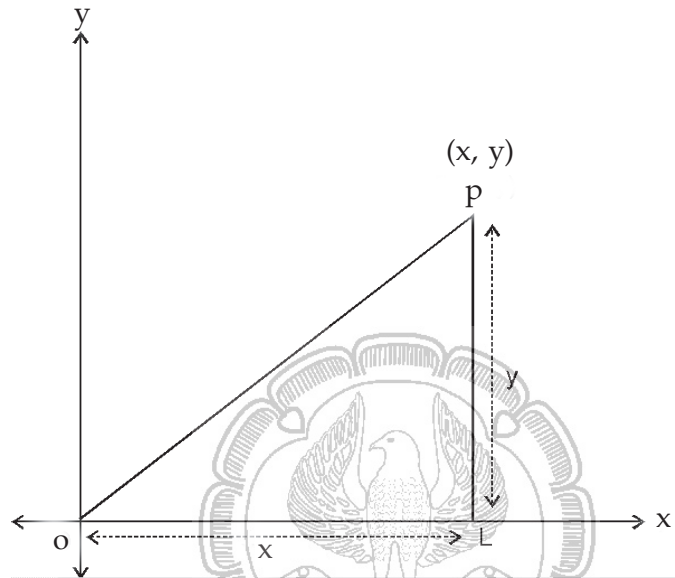
- The solution of the cubic equation $x^3 - 6x^2 + 11x - 6 = 0$ is given by the triplet :
a) (-1, 1, -2) b) (1, 2, 3) c) (-2, 2, 3) d) (0, 4, -5)
- The cubic equation $x^3 + 2x^2 - x - 2 = 0$ has 3 roots namely.
a) (1, -1, 2) b) (-1, 1, -2) c) (-1, 2, -2) d) (1, 2, 2)
- $x^4 - 4x^3 + 5x^2 - 4x + 5$ are the factors of the left-hand side of the equation.
a) $x^3 + 2x^2 - x - 2 = 0$ b) $x^3 + x^2 - 20x = 0$
c) $x^3 - 3x^2 - 4x + 12 = 0$ d) $x^3 - 6x^2 + 11x - 6 = 0$
- The equation $3x^3 + 5x^2 = 3x + 5$ has got 3 roots and hence the factors of the left-hand side of the equation $3x^3 + 5x^2 - 3x - 5 = 0$ are
a) $x - 1, x - 2, x - 5/3$ b) $x - 1, x + 1, 3x + 5$ c) $x + 1, x - 1, 3x - 5$ d) $x - 1, x + 1, x - 2$
- Factorise the left hand side of the equation $x^3 + 7x^2 - 21x - 27 = 0$ and the roots are as
a) (-3, -9, -1) b) (3, -9, -1) c) (3, 9, 1) d) (-3, 9, 1)
- The roots of $x^3 + x^2 - x - 1$ are
a) (-1, -1, 1) b) (1, 1, -1) c) (-1, -1, -1) d) (1, 1, 1)
- The satisfying value of $x^3 + x^2 - 20x = 0$ are
a) (1, 4, -5) b) (2, 4, -5) c) (0, -4, 5) d) (0, 4, -5)
- The roots of the cubic equation $x^3 + 7x^2 - 21x - 27 = 0$ are
a) (-3, -9, -1) b) (3, -9, -1) c) (3, 9, 1) d) (-3, 9, 1)
- If $4x^3 + 8x^2 - x - 2 = 0$ then value of $(2x+3)$ is given by
a) 4, -1, 2 b) -4, 2, 1 c) 2, -4, -1 d) none of these.
- The rational root of the equation $2x^3 - x^2 - 4x + 2 = 0$ is
a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 2 d) -2.



2.12 APPLICATION OF EQUATIONS IN CO - ORDINATE GEOMETRY

Introduction: Co-ordinate geometry is that branch of mathematics which explains the problems of geometry with the help of algebra

Distance of a point from the origin.

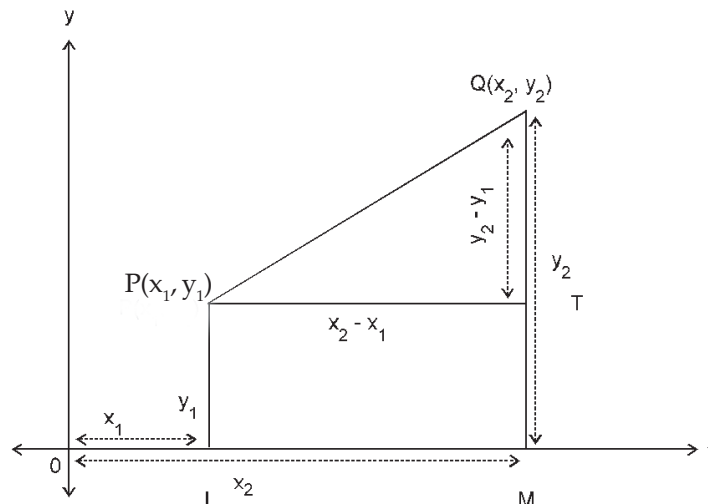


P (x, y) is a point.

By Pythagora's Theorem $OP^2 = OL^2 + PL^2$ or $OP^2 = x^2 + y^2$

So Distance OP of a point from the origin O is $\sqrt{x^2 + y^2}$

Distance between two points





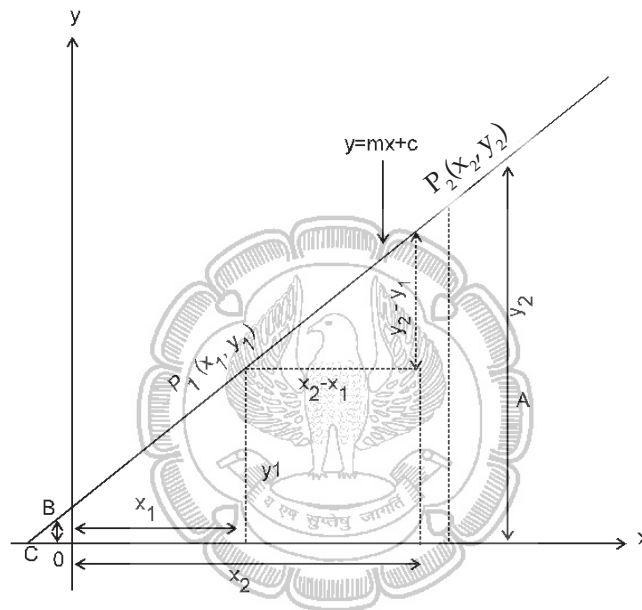
By Pythagora's Theorem $PQ^2=PT^2 +QT^2$

$$\text{or } PQ^2 = (x_2-x_1)^2 + (y_2-y_1)^2 = (x_1-x_2)^2 + (y_1-y_2)^2$$

$$\text{or } PQ = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

So distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$.

2.13 EQUATION OF A STRAIGHT LINE



- (I) The equation to a straight line in simple form is generally written as $y=mx+c$ (i) where m is called the slope and c is a constant.

If $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be any two points on the line the ratio $\frac{y_2-y_1}{x_2-x_1}$ is known as the slope of the line.

We observe that B is a point on the line $y = mx+c$ and OB is the length of the y -axis that is intercepted by the line and that for the point B $x=0$.

Substituting $x=0$ in $y=mx+c$ we find $y=c$ the intercept on the y axis.

This form of the straight line is known as slope-intercept form.

- Note : (i) If the line passes through the origin $(0, 0)$ the equation of the line becomes $y = mx$ (or $x=my$)
- (ii) If the line is parallel to x -axis, $m=0$ and the equation of the line becomes $y = c$ (or $x = b$ b is the intercept on x -axis)



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(iii) If the line coincides with x-axis, $m=0$, $c=0$ then the equation of the line becomes $y=0$ which is the equation of x-axis. Similarly $x=0$ is the equation of y-axis.

(II) Let $y = mx + c$ (i) be the equation of the line p_1p_2 .

Let the line pass through (x_1, y_1) . So we get

$$y_1 = mx_1 + c \quad \dots(ii)$$

$$\text{By (i) - (ii) } y - y_1 = m(x - x_1) \quad \dots (iii)$$

which is another form of the equation of a line to be used when the slope(m) and any point (x_1, y_1) on the line be given. This form is called **point-slope form**.

(III) If the line above line (iii) passes through another point (x_2, y_2) . we write

$$y_2 - y_1 = m(x_2 - x_1)$$

$$\text{by (iii) - (iv) } \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

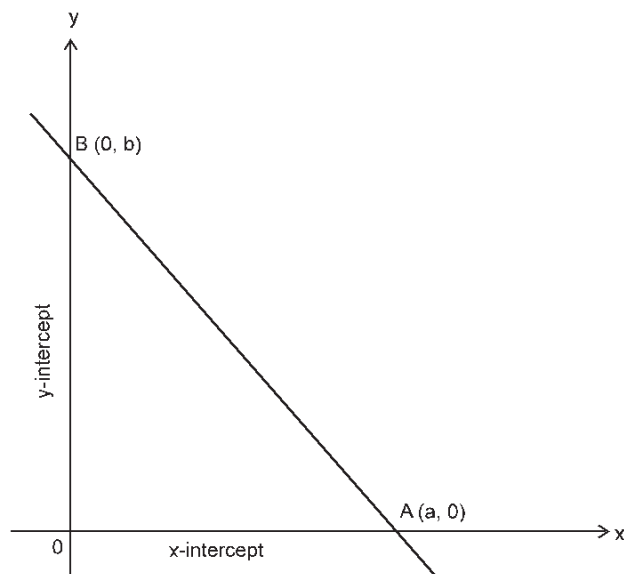
$$(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1} \right) (x - x_1)$$

Which is the equation of the line passing through two points (x_1, y_1) and (x_2, y_2)

(IV) We now consider a straight line that makes x-intercept = a and y-intercept = b

Slope of the line

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{b - 0}{0 - a} = -\frac{b}{a}$$





If (x, y) is any point on this line we may also write the slope as

$$\frac{y-0}{x-a} = \frac{y}{x-a}$$

Thus $\frac{y}{x-a} = -\frac{b}{a}$

or $\frac{y}{a} = -\frac{x-a}{a} = -\frac{x}{a} + 1$

Transposing $\frac{x}{a} + \frac{y}{b} = 1$

The form $\frac{x}{a} + \frac{y}{b} = 1$ is called intercept form of the equation of the line and the same is to be used when x -intercept and y -intercept be given.

Note: (i) The equation of a line can also be written as $ax+by+c = 0$

(ii) If we write $ax+by+c = 0$ in the form $y = mx+c$

we get $y = \left(\frac{-a}{b}\right)x + \left(\frac{-c}{a}\right)$ giving slope $m = \left(\frac{-a}{b}\right)$.

(iii) Two lines having slopes m_1 and m_2 are parallel to each other if and only if $m_1 = m_2$ and perpendicular to each other if and only if $m_1 m_2 = -1$

(iv) Let $ax + by + c = 0$ be a line. The equation of a line parallel to $ax + by + c = 0$ is $ax + by + k = 0$ and the equation of the line perpendicular to $ax + by + c = 0$ is $bx - ay + k = 0$

Let lines $ax + by + c = 0$ and $a^1x+b^1y+c^1 = 0$ intersect each other at the point (x_1, y_1) .

So $ax_1 + by_1 + c = 0$

$a^1x_1 + b^1y_1 + c^1 = 0$

By cross multiplication $\frac{x}{bc'-b^1c} = \frac{y}{ca'-ac^1} = \frac{1}{ab'-a^1b}$

$x_1 = \frac{bc'-b^1c}{ab'-a^1b}$ $y_1 = \frac{ca'-c^1a}{ab'-a^1b}$

Example : Let the lines $2x+3y+5 = 0$ and $4x-5y+2 = 0$ intersect at (x_1, y_1) . To find the point of intersection we do cross multiplication as

$2x_1 + 3y_1 + 5 = 0$

$4x_1 + 5y_1 + 2 = 0$



EQUATIONS

$$\frac{x_1}{3 \times 2 - 5 \times 5} = \frac{y_1}{5 \times 4 - 2 \times 2} = \frac{1}{2 \times 5 - 3 \times 4}$$

Solving $x_1 = 19/2$ $y_1 = -8$

(V) The equation of a line passing through the point of intersection of the lines $ax + by + c = 0$ and $a_1x + b_1y + c = 0$ can be written as $ax + by + c + K(a_1x + b_1y + c) = 0$ when K is a constant.

(VI) The equation of a line joining the points (x_1, y_1) and (x_2, y_2) is given as

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

If any other point (x_3, y_3) lies on this line we get

$$\frac{y_3 - y_1}{y_2 - y_1} = \frac{x_3 - x_1}{x_2 - x_1}$$

$$\text{or } x_2y_3 - x_2y_1 - x_1y_3 + x_1y_1 = x_3y_2 - x_3y_1 - x_1y_2 + x_1y_1 = 0$$

$$\text{or } x_1y_2 - x_1y_3 + x_2y_3 - x_2y_1 + x_3y_1 - x_3y_2 = 0$$

$$\text{or } x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

which is the required condition of collinearity of three points.

Illustrations:

1. Show that the points $A(2, 3)$ $B(4, 1)$ and $C(-2, 7)$ are collinear.

Solution : Using the rule derived in VI above we may conclude that the given points are collinear if $2(1-7) + 4(7-3) - 2(3-1) = 0$

i.e. if $-12 + 16 - 4 = 0$ which is true.

So the three given points are collinear

2. Find the equation of a line passing through the point $(5, -4)$ and parallel to the line $4x + 7y + 5 = 0$

Solution : Equation of the line parallel to $4x + 7y + 5 = 0$ is $4x + 7y + K = 0$

Since it passes through the point $(5, -4)$ we write

$$4(5) + 7(-4) + k = 0$$

$$\text{or } 20 - 28 + k = 0$$

$$\text{or } -8 + k = 0$$

$$\text{or } k = 8$$

The equation of the required line is therefore $4x + 7y + 8 = 0$.



3. Find the equation of the straight line which passes through the point of intersection of the straight lines $2x+3y = 5$ and $3x+5y = 7$ and makes equal positive intercepts on the coordinate axes.

Solution: $2x+3y-5 = 0$
 $3x+5y-7 = 0$

By cross multiplication

$$\frac{x}{-21+25} = \frac{y}{-15+14} = \frac{1}{10-9}$$

$$\text{or } \frac{x}{4} = \frac{y}{-1} = 1$$

So the point of intersection of the given lines is $(4, -1)$

Let the required equation of line be

$$\frac{x}{a} + \frac{y}{b} = 1 \text{ (*for equal positive intercepts } a=b)$$

$$\therefore x + y = a$$

Since it passes through $(4, -1)$ we get $4 - 1 = a$ or $a = 3$

The equation of the required line is therefore $x + y = 3$.

4. Prove that $(3, 1)$, $(5, -5)$ and $(-1, 13)$ are collinear and find the equation of the line through these three points.

Solution: If A $(3, 1)$ B $(5, -5)$ and C $(-1, 13)$ are collinear we may write

$$3(-5-13) + 5(13-1) - 1(1+5) = 0$$

$$\text{or } 3(-18) + 5(12) - 6 = 0 \text{ which is true.}$$

Hence the given three points are collinear.

As the points A, B, C are collinear, the required line will be the line through any of these two points. Let us find the equation of the line through B $(5, -5)$ and A $(3, 1)$

Using the rule derived in III earlier we find

$$\frac{y+5}{1+5} = \frac{x-5}{3-5} \text{ or, } \frac{y+5}{6} = \frac{x-5}{-2}$$

$$\text{or } y + 5 + 3(x - 5) = 0$$

or $3x + y = 10$ is the required line.

5. Find the equation of the line parallel to the line joining points $(7, 5)$ and $(2, 9)$ and passing through the point $(3, -4)$.



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Solution : Equation of the line through the points (7, 5) and (2, 9) is given by

$$\frac{y - 5}{9 - 5} = \frac{x - 7}{2 - 7}$$

$$\text{or } -5y + 25 = 4x - 28$$

$$\text{or } 4x + 5y - 53 = 0$$

Equation of the line parallel to $4x + 5y - 53 = 0$ is $4x + 5y + k = 0$

If it passes through (3, -4) we have $12 - 20 + k = 0$ i.e. $k = 8$

Thus the required line is $4x + 5y + 8 = 0$

6. Prove that the lines $3x - 4y + 5 = 0$, $7x - 8y + 5 = 0$ and $4x + 5y = 45$ are concurrent.

Solution: Let (x_1, y_1) be the point of intersection of the lines

$$3x - 4y + 5 = 0 \dots\dots\dots (i)$$

$$7x - 8y + 5 = 0 \dots\dots\dots (ii)$$

$$\text{Then we have } 3x_1 - 4y_1 + 5 = 0$$

$$7x_1 - 8y_1 + 5 = 0$$

$$\text{Then } \frac{x_1}{-20+40} = \frac{y_1}{35-15} = \frac{1}{-24+28} \quad \therefore x_1 = \frac{20}{4} = 5, \quad y_1 = \frac{20}{4} = 5.$$

Hence (5, 5) is the point of intersection. Now for the line $4x + 5y = 45$

we find $4(5) + 5(5) = 45$; hence (5, 5) satisfies the equation $4x + 5y = 45$.

Thus the given three lines are concurrent.

7. A manufacturer produces 80 T.V. sets at a cost Rs. 220000 and 125 T.V. sets at a cost of Rs. 287500. Assuming the cost curve to be linear find the equation of the line and then use it to estimate the cost of 95 sets.

Solution: Since the cost curve is linear we consider cost curve as $y = Ax + B$ where y is total cost. Now for $x = 80, y = 220000$. $\therefore 220000 = 80A + B \dots\dots(i)$

$$\text{and for } x = 125, y = 287500 \therefore 287500 = 125A + B \dots\dots(ii)$$

$$\text{Subtracting (i) from (ii) } 45A = 67500 \quad \text{or } A = 1500$$

$$\text{From (i) } 220000 - 1500 \times 80 = B \quad \text{or } B = 220000 - 120000 = 100000$$

Thus equation of cost line is $y = 1500x + 100000$.

$$\text{For } x = 95, y = 142500 + 100000 = \text{Rs. } 242500.$$

\therefore Cost of 95 T.V. set will be Rs. 242500.



Exercise 2(J)

Choose the most appropriate option (a) (b) (c) (d)

- The equation of line joining the point (3, 5) to the point of intersection of the lines $4x + y - 1 = 0$ and $7x - 3y - 35 = 0$ is
a) $2x - y = 1$ b) $3x + 2y = 19$ c) $12x - y - 31 = 0$ d) none of these.
- The equation of the straight line passing through the points (-5, 2) and (6, -4) is
a) $11x + 6y + 8 = 0$ b) $x + y + 4 = 0$ c) $6x + 11y + 8 = 0$ d) none of these
- The equation of the line through (-1, 3) and parallel to the line joining (6, 3) and (2, -3) is
a) $3x - 2y + 9 = 0$ b) $3x + 2y - 7 = 0$ c) $x + y - 7 = 0$ d) none of these
- The equation of a straight line passing through the point (-2, 3) and making intercepts of equal length on the axes is
(a) $2x + y + 1 = 0$ b) $x - y + 5 = 0$ c) $x - y + 5 = 0$ d) $x + y - 1 = 0$
- If the lines $3x - 4y - 13 = 0$, $8x - 11y - 33 = 0$ and $2x - 3y + \lambda = 0$ are concurrent then value of λ is
(a) 11 (b) 5 (c) -7 (d) none of these
- The total cost curve of the number of copies of a particular photograph is linear. The total cost of 5 and 8 copies of a photograph are Rs.80 and Rs.116 respectively. The total cost for 10 copies of the photograph will be
(a) Rs. 100 (b) Rs. 120 (c) Rs. 120 (d) Rs. 140
- A firm produces 50 units of a product for Rs.320 and 80 units for Rs.380. Considering the cost curve to be a straight-line the cost of producing 110 units to be estimated as
(a) 400 (b) 420 (c) 440 (d) none of these.
- The total cost curve of the number of copies photograph is linear. The total cost of 5 and 18 copies of a photographs are Rs.80 and 116 respectively. Then the total cost for 10 copies of the photographs is
(a) Rs. 140 (b) 93.85 (c) 150 (d) Rs. 130

2.14 GRAPHICAL SOLUTION TO LINEAR EQUATIONS

- Drawing graphs of straight lines

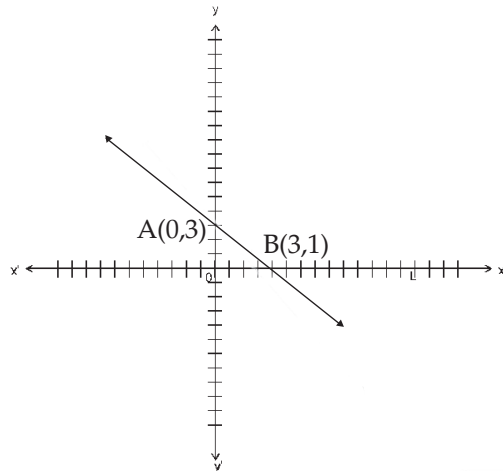
From the given equation we tabulate values of (x, y) at least 2 pairs of values and then plot them in the graph taking two perpendicular axis (x, y axis). Then joining the points we get the straight line representing the given equation.

Example1 : Find the graph of the straight line having equation $3y = 9 - 2x$



Solution: We have $2x + 3y = 9$. We tabulate $y = \frac{9 - 2x}{3}$

x	0	3
y	3	1



Here AB is the required straight line shown in the graph.

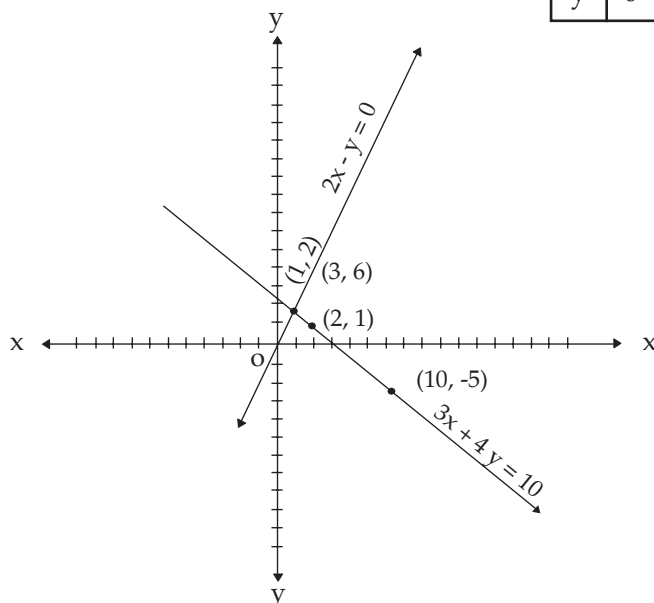
Example 2 : Draw graph of the straight lines $3x + 4y = 10$ and $2x - y = 0$ and find the point of intersection of these lines.

Solution: For $3x + 4y = 10$ we have $y = \frac{10 - 3x}{4}$; we tabulate

x	2	10	-10
y	1	-5	10

For $2x - y = 0$ we tabulate

x	0	1	3
y	0	2	6





From the graph, the point of intersection is (1, 2)

Exercise (2K)

Choose the most appropriate option (a) (b) (c) (d)

1. A right angled triangle is formed by the straight line $4x+3y=12$ with the axes. Then length of perpendicular from the origin to the hypotenuse is
(a) 3.5 units (b) 2.4 units (c) 4.2 units (d) none of these.
2. The distance from the origin to the point of intersection of two straight lines having equations $3x-2y=6$ and $3x+2y=18$ is
(a) 3 units (b) 5 units (c) 4 units (d) 2 units.
3. The point of intersection between the straight lines $3x + 2y = 6$ and $3x - y = 12$ lie in
(a) 1st quadrant (b) 2nd quadrant (c) 3rd quadrant (d) 4th quadrant.



**EQUATIONS****ANSWERS****Exercise 2(A)**

1. b	2. a	3. c	4. c	5. b	6. d	7. a	8. d
9. c							

Exercise 2(B)

1. c	2. b	3. a	4. b	5. c	6. a	7. d	8. d
9. a	10. c	11. c	12. a				

Exercise 2(C)

1. b	2. c	3. a	4. a	5. d	6. a	7. b	8. c
9. b	10. d						

Exercise 2(D)

1. a	2. c	3. a	4. d	5. a	6. c	7. a	8. c
9. b	10. d						

Exercise 2(E)

1. b	2. a	3. d	4. c	5. b	6. c	7. a	8. a
9. c	10. b	11. a					

Exercise 2(F)

1. d	2. d	3. b	4. b	5. a	6. c	7. d	8. b
9. a	10. b	11. d	12. c	13. b			

Exercise 2(G)

1. b	2. d	3. a	4. d	5. b	6. b	7. a	8. c
9. c	10. a						

Exercise 2(H)

1. c	2. b	3. a	4. b	5. c	6. d	7. a	8. b
9. c	10. a	11. a	12. c				

Exercise 2(I)

1. c	2. b	3. b	4. b	5. b	6. a	7. d	8. b
9. a	10. c						

Exercise 2(J)

1. c	2. c	3. a	4. d	5. c	6. d	7. c	8. b
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Exercise 2(K)

1. b	2. b	3. d					
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ADDITIONAL QUESTION BANK

- Solving equation $x^2 - (a+b)x + ab = 0$ are, value(s) of x
(A) a, b (B) a (C) b (D) None
- Solving equation $x^2 - 24x + 135 = 0$ are, value(s) of x
(A) 9, 6 (B) 9, 15 (C) 15, 6 (D) None
- If $\frac{x}{b} + \frac{b}{x} = \frac{a}{b} + \frac{b}{a}$ the roots of the equation are
(A) $a, b^2/a$ (B) $a^2, b/a^2$ (C) $a^2, b^2/a$ (D) a, b^2
- Solving equation $\frac{6x+2}{4} + \frac{2x^2-1}{2x^2+2} = \frac{10x-1}{4x}$ we get roots as
(A) ± 1 (B) $+1$ (C) -1 (D) 0
- Solving equation $3x^2 - 14x + 16 = 0$ we get roots as
(A) ± 1 (B) 2 and $\frac{8}{3}$ (C) 0 (D) None
- Solving equation $3x^2 - 14x + 8 = 0$ we get roots as
(A) ± 4 (B) ± 2 (C) 4, $\frac{2}{3}$ (D) None
- Solving equation $(b-c)x^2 + (c-a)x + (a-b) = 0$ following roots are obtained
(A) $\frac{a-b}{b-c}, 1$ (B) $(a-b)(a-c), 1$ (C) $\frac{b-c}{a-b}, 1$ (D) None
- Solving equation $7\sqrt{\frac{x}{1-x}} + 8\sqrt{\frac{1-x}{x}} = 15$ following roots are obtained
(A) $\frac{49}{50}, \frac{64}{65}$ (B) $\frac{1}{50}, \frac{1}{65}$ (C) $\frac{49}{50}, \frac{1}{65}$ (D) $\frac{1}{50}, \frac{64}{65}$
- Solving equation $6 \left[\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} \right] = 15$ following roots are obtained
(A) $\frac{4}{13}, \frac{9}{13}$ (B) $\frac{-4}{13}, \frac{-9}{13}$ (C) $\frac{4}{13}, \frac{5}{13}$ (D) $\frac{6}{13}, \frac{7}{13}$
- Solving equation $z^2 - 6z + 9 = 4\sqrt{z^2 - 6z + 6}$ following roots are obtained
(A) $3+2\sqrt{3}, 3-2\sqrt{3}$ (B) 5, 1 (C) all the above (D) None



EQUATIONS

11. Solving equation $\frac{x+\sqrt{12p-x}}{x-\sqrt{12p-x}} = \frac{\sqrt{p+1}}{\sqrt{p-1}}$ following roots are obtained
(A) $3p$ (B) both $3p$ and $-4p$ (C) only $-4p$ (D) $-3p$ $4p$
12. Solving equation $(1+x)^{2/3} + (1-x)^{2/3} = 4(1-x^2)^{1/3}$ are, values of x
(A) $\frac{5}{\sqrt{3}}$ (B) $-\frac{5}{\sqrt{3}}$ (C) $\pm \frac{5}{3\sqrt{3}}$ (D) $\pm \frac{15}{\sqrt{3}}$
13. Solving equation $(2x+1)(2x+3)(x-1)(x-2)=150$ the roots available are
(A) $\frac{1\pm\sqrt{129}}{4}$ (B) $\frac{7}{2}, -3$ (C) $-\frac{7}{2}, 3$ (D) None
14. Solving equation $(2x+3)(2x+5)(x-1)(x-2)=30$ the roots available are
(A) $0, \frac{1}{2}, -\frac{11}{4}, \frac{9}{4}$ (B) $0, -\frac{1}{2}, \frac{-1\pm\sqrt{105}}{4}$ (C) $0, -\frac{1}{2}, -\frac{11}{4}, -\frac{9}{4}$ (D) None
15. Solving equation $z + \sqrt{z} = \frac{6}{25}$ the value of z works out to
(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{1}{25}$ (D) $\frac{2}{25}$
16. Solving equation $z^{10} - 33z^5 + 32 = 0$ the following values of z are obtained
(A) $1, 2$ (B) $2, 3$ (C) $2, 4$ (D) $1, 2, 3$
17. When $\sqrt{2z+1} + \sqrt{3z+4} = 7$ the value of z is given by
(A) 1 (B) 2 (C) 3 (D) 4
18. Solving equation $\sqrt{x^2-9x+18} + \sqrt{x^2-2x-15} = \sqrt{x^2-4x+3}$ following roots are obtained
(A) $3, \frac{2\pm\sqrt{94}}{3}$ (B) $\frac{2\pm\sqrt{94}}{3}$ (C) $4, -\frac{8}{3}$ (D) $3, 4, -\frac{8}{3}$
19. Solving equation $\sqrt{y^2+4y-21} + \sqrt{y^2-y-6} = \sqrt{6y^2-5y-39}$ following roots are obtained
(A) $2, 3, 5/3$ (B) $2, 3, -5/3$ (C) $-2, -3, 5/3$ (D) $-2, -3, -5/3$
20. Solving equation $6x^4 + 11x^3 - 9x^2 - 11x + 6 = 0$ following roots are obtained
(A) $\frac{1}{2}, -2, \frac{-1\pm\sqrt{37}}{6}$ (B) $-\frac{1}{2}, 2, \frac{-1\pm\sqrt{37}}{6}$ (C) $\frac{1}{2}, -2, \frac{5}{6}, \frac{-7}{6}$ (D) None



21. If $\frac{x-bc}{d+c} + \frac{x-ca}{c+a} + \frac{x-ab}{a+b} = a+b+c$ the value of x is
(A) $a^2+b^2+c^2$ (B) $a(a+b+c)$ (C) $(a+b)(b+c)$ (D) $ab+bc+ca$
22. If $\frac{x+2}{x-2} - \frac{x-2}{x+2} = \frac{x-1}{x+3} - \frac{x+3}{x-3}$ then the values of x are
(A) $0, \pm\sqrt{6}$ (B) $0, \pm\sqrt{3}$ (C) $0, \pm 2\sqrt{3}$ (D) None
23. If $\frac{x-a}{b} + \frac{x-b}{a} = \frac{b}{x-a} + \frac{a}{x-b}$ then the values of x are
(A) $0, (a+b)(a-b)$ (B) $0, (a+b), \frac{a^2+b^2}{a+b}$ (C) $0, (a-b), \frac{a^2+b^2}{a+b}$ (D) $\frac{a^2+b^2}{a+b}$
24. If $\frac{x-a^2-b^2}{c^2} + \frac{c^2}{x-a^2-b^2} = 2$ the value of x is
(A) $a^2+b^2+c^2$ (B) $-a^2-b^2-c^2$ (C) $\frac{1}{a^2+b^2+c^2}$ (D) $-\frac{1}{a^2+b^2+c^2}$
25. Solving equation $\left(x - \frac{1}{x}\right)^2 - 6\left(x + \frac{1}{x}\right) + 12 = 0$ we get roots as follows
(A) 0 (B) 1 (C) -1 (D) None
26. Solving equation $\left(x - \frac{1}{x}\right)^2 - 10\left(x - \frac{1}{x}\right) + 24 = 0$ we get roots as follows
(A) 0 (B) 1 (C) -1 (D) $(2 \pm \sqrt{5}), (3 \pm \sqrt{10})$
27. Solving equation $2\left(x - \frac{1}{x}\right)^2 - 5\left(x + \frac{1}{x} + 2\right) + 18 = 0$ we get roots as under
(A) 0 (B) 1 (C) -1 (D) $-2 \pm \sqrt{3}$
28. If α, β are the roots of equation $x^2 - 5x + 6 = 0$ the equation with roots $(\alpha + \beta)$ and $(\alpha - \beta)$ is
(A) $x^2 - 6x + 5 = 0$ (B) $2x^2 - 6x + 5 = 0$ (C) $2x^2 - 5x + 6 = 0$ (D) $x^2 - 5x + 6 = 0$
29. If α, β are the roots of equation $x^2 - 5x + 6 = 0$ the equation with roots $(\alpha^2 + \beta)$ and $(\alpha + \beta^2)$ is
(A) $x^2 - 9x + 99 = 0$ (B) $x^2 - 18x + 90 = 0$ (C) $x^2 - 18x + 77 = 0$ (D) None
30. If α, β are the roots of equation $x^2 - 5x + 6 = 0$ the equation with roots $(\alpha\beta + \alpha + \beta)$ and $(\alpha\beta - \alpha - \beta)$ is
(A) $x^2 - 12x + 11 = 0$ (B) $2x^2 - 6x + 12 = 0$ (C) $x^2 - 12x + 12 = 0$ (D) None



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31. The condition that one of $ax^2+bx+c=0$ the roots of is twice the other is
(A) $b^2=4ca$ (B) $2b^2=9(c+a)$ (C) $2b^2=9ca$ (D) $2b^2=9(c-a)$
32. The condition that one of $ax^2+bx+c=0$ the roots of is thrice the other is
(A) $3b^2=16ca$ (B) $b^2=9ca$ (C) $3b^2=-16ca$ (D) $b^2=-9ca$
33. If the roots of $ax^2+bx+c=0$ are in the ratio $\frac{p}{q}$ then the value of $\frac{b^2}{ca}$ is
(A) $\frac{(p+q)^2}{(pq)}$ (B) $\frac{(p+q)}{(pq)}$ (C) $\frac{(p-q)^2}{(pq)}$ (D) $\frac{(p-q)}{(pq)}$
34. Solving $6x+5y-16=0$ and $3x-y-1=0$ we get values of x and y as
(A) 1, 1 (B) 1, 2 (C) -1, 2 (D) 0, 2
35. Solving $x^2+y^2-25=0$ and $x-y-1=0$ we get the roots as under
(A) $\pm 3 \pm 4$ (B) $\pm 2 \pm 3$ (C) 0, 3, 4 (D) 0, -3, -4
36. Solving $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} - \frac{5}{2} = 0$ and $x+y-5=0$ we get the roots as under
(A) 1, 4 (B) 1, 2 (C) 1, 3 (D) 1, 5
37. Solving $\frac{1}{x^2} + \frac{1}{y^2} - 13 = 0$ and $\frac{1}{x} + \frac{1}{y} - 5 = 0$ we get the roots as under
(A) $\frac{1}{8}, \frac{1}{5}$ (B) $\frac{1}{2}, \frac{1}{3}$ (C) $\frac{1}{13}, \frac{1}{5}$ (D) $\frac{1}{4}, \frac{1}{5}$
38. Solving $x^2+xy-21=0$ and $xy-2y^2+20=0$ we get the roots as under
(A) $\pm 1, \pm 2$ (B) $\pm 2, \pm 3$ (C) $\pm 3, \pm 4$ (D) None
39. Solving $x^2+xy+y^2=37$ and $3xy+2y^2=68$ we get the following roots
(A) $\pm 3 \pm 4$ (B) $\pm 4 \pm 5$ (C) $\pm 2 \pm 3$ (D) None
40. Solving $4^x \cdot 2^y = 128$ and $3^{3x+2y} = 9^{xy}$ we get the following roots
(A) $\frac{7}{4}, \frac{7}{2}$ (B) 2, 3 (C) 1, 2 (D) 1, 3
41. Solving $9^x = 3^y$ and $5^{x+y+1} = 25^{xy}$ we get the following roots
(A) 1, 2 (B) 0, 1 (C) 0, 3 (D) 1, 3



42. Solving $9x+3y-4z=3$ $x+y-z=0$ and $2x-5y-4z=-20$ following roots are obtained
(A) 2, 3, 4 (B) 1, 3, 4 (C) 1, 2, 3 (D) None
43. Solving $x+2y+2z=0$ $3x-4y+z=0$ and $x^2+3y^2+z^2=11$ following roots are obtained
(A) 2, 1, -2 and -2, -1, 2 (B) 2, 1, 2 and -2, -1, -2
(C) only 2, 1, -2 (D) only -2, -1, 2
44. Solving $x^3-6x^2+11x-6=0$ we get the following roots
(A) -1, -2, 3 (B) 1, 2, -3 (C) 1, 2, 3 (D) -1, -2, -3
45. Solving $x^3+9x^2-x-9=0$ we get the following roots
(A) $\pm 1, -9$ (B) $\pm 1, \pm 9$ (C) $\pm 1, 9$ (D) None
46. It is being given that one of the roots is half the sum of the other two solving $x^3-12x^2+47x-60=0$ we get the following roots:
(A) 1, 2, 3 (B) 3, 4, 5 (C) 2, 3, 4 (D) -3, -4, -5
47. Solve $x^3+3x^2-x-3=0$ given that the roots are in arithmetical progression
(A) -1, 1, 3 (B) 1, 2, 3 (C) -3, -1, 1 (D) -3, -2, -1
48. Solve $x^3-7x^2+14x-8=0$ given that the roots are in geometrical progression
(A) $\frac{1}{2}, 1, 2$ (B) 1, 2, 4 (C) $\frac{1}{2}, -1, 2$ (D) -1, 2, -4
49. Solve $x^3-6x^2+5x+12=0$ given that the product of the two roots is 12
(A) 1, 3, 4 (B) -1, 3, 4 (C) 1, 6, 2 (D) 1, -6, -2
50. Solve $x^3-5x^2-2x+24=0$ given that two of its roots being in the ratio of 3:4
(A) -2, 4, 3 (B) -1, 4, 3 (C) 2, 4, 3 (D) -2, -4, -3
51. The points (-3, 4), (2, 4) and (1, 2) are the vertices of a triangle which is
(A) right angled (B) isosceles (C) equilateral (D) other
52. The points (2, 3), (-5, 2) and (-6, -9) are the vertices of a triangle which is
(A) right angled (B) isosceles (C) equilateral (D) other
53. The points (2, 3), (-5, 2) and (-4, 9) are the vertices of a triangle which is
(A) right angled (B) isosceles (C) equilateral (D) other
54. The points (2, 7), (5, 3) and (-2, 4) are the vertices of a triangle which is
(A) right angled (B) isosceles (C) equilateral (D) other
55. The points (1, -1), $(-\sqrt{3}, -\sqrt{3})$ and (-1, 1) are the vertices of a triangle which is
(A) right angled (B) isosceles (C) equilateral (D) other



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56. The points $(2, -1)$ $(-2, 3)$ $(3, 4)$ and $(-3, -2)$ are the vertices of a
(A) square (B) rhombus (C) parallelogram (D) rectangle
57. The points $(\frac{1}{2}, -\sqrt{\frac{3}{2}})$ $(-\sqrt{\frac{3}{2}}, \frac{1}{2})$ $(-\frac{1}{2}, -\sqrt{\frac{3}{2}})$ and $(\sqrt{\frac{3}{2}}, -\frac{1}{2})$ are the vertices of a triangle which is
(A) square (B) rhombus (C) parallelogram (D) rectangle
58. The points $(2, -2)$ $(-1, 1)$ $(8, 4)$ and $(5, 7)$ are the vertices of a
(A) square (B) rhombus (C) parallelogram (D) rectangle
59. The points $(2, 1)$ $(3, 3)$ $(5, 2)$ and $(6, 4)$ are the vertices of a
(A) square (B) rhombus (C) parallelogram (D) rectangle
60. The co-ordinates of the circumcentre of a triangle with vertices $(3, -2)$ $(-6, 5)$ and $(4, 3)$ are
(A) $(-\frac{3}{2}, \frac{3}{2})$ (B) $(\frac{3}{2}, -\frac{3}{2})$ (C) $(-3, 3)$ (D) $(3, -3)$
61. The centroid of a triangle with vertices $(1, -2)$ $(-5, 3)$ and $(7, 2)$ is given by
(A) $(0, 0)$ (B) $(1, -1)$ (C) $(-1, 1)$ (D) $(1, 1)$
62. The ratio in which the point $(11, -3)$ divides the joint of points $(3, 4)$ and $(7, 11)$ is
(A) 1:1 (B) 2:1 (C) 3:1 (D) None
63. The area of a triangle with vertices $(1, 3)$ $(5, 6)$ and $(-3, 4)$ in terms of square units is
(A) 5 (B) 3 (C) 8 (D) 13
64. The area of a triangle with vertices $(0, 0)$ $(1, 2)$ and $(-1, 2)$ is
(A) 2 (B) 3 (C) 1 (D) None
65. The area of the triangle bounded by the lines $4x+3y+8=0$ $x-y+2=0$ and $9x-2y-17=0$ is
(A) 18 (B) 17.5 (C) 17 (D) None
66. The area of the triangle with vertices $(4, 5)$ $(1, -1)$ and $(2, 1)$ is
(A) 0 (B) 1 (C) -1 (D) None
67. The area of the triangle with vertices $(-3, 16)$ $(3, -2)$ and $(1, 4)$ is
(A) 0 (B) 1 (C) -1 (D) None
68. The area of the triangle with vertices $(-1, 1)$ $(3, -2)$ and $(-5, 4)$ is
(A) 0 (B) 1 (C) -1 (D) None
69. The area of the triangle with vertices $(p, q+r)$ $(q, r+p)$ and $(r, p+q)$ is
(A) 0 (B) 1 (C) -1 (D) None



70. The area of the quadrilateral with vertices (1, 7) (3, -5) (6, -2) and (-4, 2) is
(A) 50 (B) 55 (C) 56 (D) 57
71. The centroid of the triangle with vertices $(p-q, p-r)$ $(q-r, q-p)$ and $(r-p, r-q)$ is located at
(A) (1, 1) (B) (-1, 1) (C) (1, -1) (D) the origin
72. A lotus over a pond is 1" above the water level. With cool breeze it immersed 7" apart. The depth of the pond in terms of inches is
(A) 25 (B) 24 (C) 26 (D) None
73. Points $(p, 0)$ $(0, q)$ and (1, 1) are collinear if
(A) $\frac{1}{p} + \frac{1}{q} = 1$ (B) $\frac{1}{p} - \frac{1}{q} = 1$ (C) $\frac{1}{p} + \frac{1}{q} = 0$ (D) $\frac{1}{p} - \frac{1}{q} = 0$
74. The gradient or slope of the line where the line subtends an angle q with the X-axis is
(A) $\sin \theta$ (B) $\cos \theta$ (C) $\tan \theta$ (D) $\operatorname{cosec} \theta$
75. The equation of the line passing through (5, -3) and parallel to the line is
(A) $2x-3y+19=0$ (B) $2x-3y-14=0$ (C) $3x+2y-19=0$ (D) $3x+2y+14=0$
76. The equation of the line passing through (5, -3) and perpendicular to the line $2x-3y+14=0$ is
(A) $3x+2y-9=0$ (B) $3x+2y+14=0$ (C) $2x-3y-9=0$ (D) $2x-3y-14=0$
77. The orthocenter of the triangle bound by lines $3x-y=9$ $x-y=5$ and $2x-y=8$ is
(A) (0, 0) (B) (-6, 1) (C) (6, -1) (D) (-6, -1)
78. The equation of the line passing through points (1, -1) and (-2, 3) is given by
(A) $4x+3y-1=0$ (B) $4x+3y+1=0$ (C) $4x-3y-1=0$ (D) $4x-3y+1=0$
79. The equation of the line passing through (2, -2) and the point of intersection of $2x+3y-5=0$ and $7x-5y-2=0$ is
(A) $3x-y-4=0$ (B) $3x+y-4=0$ (C) $3x+y+4=0$ (D) None
80. The equation of the line passing through the point of intersection of $2x+3y-5=0$ and $7x-5y-2=0$ and parallel to the lines $2x-3y+14=0$ is
(A) $2x-3y+1=0$ (B) $2x-3y-1=0$ (C) $3x+2y+1=0$ (D) $3x+2y-1=0$
81. The equation of the line passing through the point of intersection of $2x+3y-5=0$ and $7x-5y-2=0$ and perpendicular to the lines $2x-3y+14=0$ is
(A) $3x+2y+5=0$ (B) $3x+2y-5=0$ (C) $2x-3y+5=0$ (D) $2x-3y-5=0$



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82. The lines $x-y-6=0$, $6x+5y+8=0$ and $4x-3y-20=0$ are
(A) Concurrent (B) Non Concurrent
(C) Perpendicular to each other (D) Parallel to each other
83. The lines $2x-y-3=0$, $3x-2y-1=0$ and $x-3y+2=0$ are
(A) Concurrent (B) Non Concurrent
(C) Perpendicular to each other (D) Parallel to each other
84. The triangle bound by the lines $y=0$, $\sqrt{3}x+y-2=0$ and $\sqrt{3}x-y+1=0$ is
(A) right angled (B) isosceles (C) equilateral (D) other
85. The equation of the line passing through $(-1, 1)$ and subtending an angle of 45° with the line $6x+5y-1=0$ is
(A) $x+11y-10=0$ (B) $11x-y+12=0$ (C) both the above (D) None
86. The equation of the line passing through $(-1, 1)$ and subtending an angle of 60° with the line $\sqrt{3}x+y-1=0$ is
(A) $y-1=0$ (B) $\sqrt{3}x-y+(\sqrt{3}+1)$ (C) both the above (D) None
87. The line joining $(-8, 3)$ and $(2, 1)$ and the line joining $(6, 0)$ and $(11, -1)$ are
(A) perpendicular (B) parallel
(C) concurrent (D) intersecting to each other at the angle of 45°
88. The line joining $(-1, 1)$ and $(2, -2)$ and the line joining $(1, 2)$ and $(2, k)$ are parallel to each other for the following value of k
(A) 1 (B) 0 (C) -1 (D) None
89. The equation of the second line in question No. (88) is
(A) $x+y+3=0$ (B) $x+y+1=0$ (C) $x+y-3=0$ (D) $x+y-1=0$
90. The line joining $(-1, 1)$ and $(2, -2)$ and the line joining $(1, 2)$ and $(2, k)$ are perpendicular to each other for the following value of k
(A) 1 (B) 0 (C) -1 (D) 3
91. The equation of the second line in question No. (90) is
(A) $x-y-1=0$ (B) $x-y+1=0$ (C) $x-y-3=0$ (D) $x-y+3=0$
92. A factory produces 300 units and 900 units at a total cost of Rs.6800/- and Rs.10400/- respectively. The linear equation of the total cost line is
(A) $y=6x+1,000$ (B) $y=5x+5,000$ (C) $y=6x+5,000$ (D) None



93. If in question No. (92) the selling price is Rs.8/- per unit the break-even point will arise at the level of _____units.
(A) 1500 (B) 2000 (C) 2500 (D) 3000
94. If instead in terms of question No. (93) if a profit of Rs.2000/- is to be earned sale and production levels have to be elevated to _____units.
(A) 3000 (B) 3500 (C) 4000 (D) 3700
95. If instead in terms of question No. (93) if a loss of Rs.3000/- is budgeted the factory may maintain production level at _____units.
(A) 1000 (B) 1500 (C) 1800 (D) 2000
96. A factory produces 200 bulbs for a total cost of Rs.800/- and 400 bulbs for Rs.1200/-. The equation of the total cost line is
(A) $2x-y+100=0$ (B) $2x+y+400=0$ (C) $2x-y+400=0$ (D) None
97. If in terms of question No.(96) the factory intends to produce 1000 bulbs the total cost would be Rs._____.
(A) 1400 (B) 1200 (C) 1300 (D) 1100
98. If an investment of Rs.1000 and Rs.100 yield an income of Rs.90 Rs.20 respectively for earning Rs.50 investment of Rs._____ will be required.
(A) less than Rs.500 (B) over Rs.500 (C) Rs.485 (D) Rs.486
99. The equation in terms of question No.(98) is
(A) $7x - 9y + 1100 = 0$ (B) $7x - 90y + 1000 = 0$
(C) $7x - 90y + 1100 = 0$ (D) $7x - 90y - 1100 = 0$
100. If an investment of Rs.60000 and Rs.70000 respectively yields an income of Rs.5750 Rs.6500 an investment of Rs.90000 would yield income of Rs._____.
(A) 7500 (B) 8000 (C) 7750 (D) 7800
101. In terms of question No.(100) an investment of Rs.50000 would yield income of Rs._____.
(A) exactly 5000 (B) little over 5000 (C) little less than 5000 (D) at least 6000
102. The equation in terms of question No.(100) is
(A) $3x + 40y + 25,000 = 0$ (B) $3x - 40y + 50,000 = 0$
(C) $3x - 40y + 25,000 = 0$ (D) $3x - 40y - 50,000 = 0$



EQUATIONS

ANSWERS

1)	A	18)	A	35)	A	52)	D	69)	A	86)	C
2)	B	19)	B	36)	A	53)	B	70)	C	87)	B
3)	A	20)	A	37)	B	54)	A, B	71)	D	88)	A
4)	A	21)	D	38)	C	55)	C	72)	B	89)	C
5)	B	22)	D	39)	A	56)	B	73)	A	90)	D
6)	C	23)	D	40)	A	57)	A	74)	C	91)	B
7)	A	24)	A	41)	A	58)	D	75)	A	92)	C
8)	A	25)	B	42)	C	59)	C	76)	A	93)	C
9)	A	26)	D	43)	A	60)	A	77)	B	94)	B
10)	C	27)	D	44)	C	61)	D	78)	A	95)	A
11)	A	28)	A	45)	A	62)	B	79)	B	96)	C
12)	C	29)	C	46)	B	63)	C	80)	A	97)	A
13)	A	30)	A	47)	C	64)	A	81)	B	98)	D
14)	B	31)	C	48)	B	65)	B	82)	A	99)	C
15)	C	32)	A	49)	B	66)	A	83)	B	100)	B
16)	A	33)	A	50)	A	67)	A	84)	C	101)	A
17)	D	34)	B	51)	D	68)	A	85)	C	102)	B